Dynamics of Arms Race Model at Fractional Order

Yoothana Suansook
Defence Technology Institute
Nonthaburi, Thailand
Yoothana.s@dti.or.th

***Abstract*—Mathematical theory of fractional calculus has applied to study many fields of physical science recently. In this paper, this theory has applied further to study on mathematical model of arms race proposed by Saperstein. The model is in the discrete form of couple logistic equations. This type of dynamical system may provide key to understand complex behaviors in social and political science in term of nonlinear dynamics. The conflict event that leads to outbreak of war can be identified as the transition to chaotic state in dynamical system. The dynamical behaviors of the arms race model explain by phase space. The numerical calculations of the arms race model have presented their fractional-order state behaviors.**

Keywords—fractional calculus; chaotic dynamics; arms race;

I. INTRODUCTION

The concept of fractional calculus was a note from Leibniz to L'Hospital about the haft-order derivative in 1695.Theory is a generalization of integer order integral and differential calculus [6]. The theory has invented for more than 300 years, but is not known much to engineering until recently [5][6]. The theory have started to be interesting to scientist and engineering when certain physical system is more accurate described by fractional derivative [6]. Many physical systems are better formulated with fractional calculus include the diffusion phenomena in inhomogeneous media with non-integer derivative; the fractional derivatives model of viscoelastic material; fractional order impedance in electric circuit; dynamical process of heat conduction and chaotic dynamical system [5][6]. Mathematical operators such as Lagrangian and Hamiltonian can be reformulated to include fractional order derivatives. This leads directly to equations of motion with nonconservative forces such as friction [35]. Oldham and Spanier and Podlubny have provided the comprehensive discussion of the theory [5][6].

The complex behavior often arises in nonlinear dynamical system. The mathematical modeling is often feasible to describe the complex behavior in nonlinear dynamical system. The mathematical model of arms race between two nations is fundamental idea in studies conflict since the Cold War period [20]. The modeling of conflicts situations are often describe by game theory. In some case the security condition change in game interaction could trigger the instability [25]. The mathematical modeling of this kind of situation is able to model by game theory where the players of the game are participate nations [25].

The fundamental question for the cause of conflict between or among nations that lead to violence situation or outbreak of war are often focus on the armament power [22]; where the competition in higher level in military areas or arms race serves as major cause of war [22].The securities of the nation are often relied on the military power. The competition between two rivalry nations in order to gain superior in military power and armament technology is considered as arms race [20]. The arms race situation can recognize by the high growth rates of armament expenditure [20]. The sources of crisis in many countries often have the cause from escalation of the growth in armament and confrontation between countries. The single event would trigger the large consequence results, i.e. the cause of World War I was started from single assassination of the Archduke Francis Ferdinand at Sarajevo in 1917 [26].

The study of complex behavior in nonlinear dynamical system has been intensively studied since the discovery chaotic pattern in weather model by Lorenz and is known as chaos theory [1]. Theory explains that certain nonlinear system behaviors are unpredictable in the long period. This theory has applied to study numerous fields of physical science. Theory of chaos explains that small change in dynamical system can cause large consequence results [17]. Theory has applied to study arms race model by many author include Saperstein, Grossman and Mayer-Kress [20]. They have postulated that political crisis or outbreak of war is associated with the onset of instability or divergent solutions rather than the loss of predictability [22].

The study of conflict between nations was formulated as mathematical model of arms race by Lewis Richardson since World War I [22]. The model explains the complex behavior arises from economics situation and competition in armament expeditions. The model is in linear differential form of arms race between two nations [22].The conflicts that lead to chaotic events such as outbreak of war are far more complicate than solution of linear differential equation [22].

In this context the analysis of mathematical properties of bounded chaos could be used as an indicator of an impending instability [22]. In this paper, we aim to apply mathematical theory of fractional calculus to study further on the variation form of Saperstein arms race model. The numerical results are feasible to provide useful interpretation in mathematical modeling. The computational results are feasible to characterize the chaotic property in different regime.

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