## The Finite Element Beam Propagation Method Based on Adaptive Padé Approximation for Analyzing a Wide-Angle Waveguide

Marinda Hongthong<sup>1,2</sup> and Tuptim Angkaew<sup>2</sup>

<sup>1</sup>Defence Technology Institute (Public Organization)

Ministry of Defence, Pak Kret, Nonthaburi 11120, Thailand

<sup>2</sup>Department of Electrical Engineering, Faculty of Engineering

Chulalongkorn University, Pathumwan, Bangkok 10330, Thailand

E-mail: marinda.h@dti.or.th, tuptim.a@chula.ac.th

Abstract—This paper proposed a mathematical model for wide-angle structure to analyze and compare results from the first and second order of Padé approximation. The proposed method aims to improve accuracy and efficiency of the Vectorial Finite Element Beam Propagation Method (FE-VBPM) using the Padé approximation in the wide-angle formulation of the waveguide structure. The computational complexity is greatly reduced by using the transverse FE-BPM with rectangular nodal elements. The boundary condition includes the 2D perfectly matched layer (PML) in its calculation. The proposed method studies wave equations in the frequency domain of an isotropic medium. The relative power is calculated in the tapered waveguide, which is the non-uniform structure formulation. The simulation results are obtained and analyzed on four different angles of tapered waveguide structure.

Keywords—Finite Element Method, Beam Propagation Method, Padé approximation, Perfectly Matched Layer, tapered waveguide

## I. INTRODUCTION

The large number of users in telecommunication system increases information exchanging demand. So, the telecommunication technology has to be developed in order to achieve user satisfactory in terms of speed, capacity, security and etc. The speed of light is crucial for communication technology especially in optical waveguide applications. The users can therefore access, download and exchange their information with high speed rate. In the past, many researches had improved performance of the optical waveguides by developing the Optical Integrated Circuit (OIC) or Photonic Integrated Circuit (PIC) [1]. OIC or PIC combines many different types of optical waveguide within a single small device using optical transmission instead of electrical transmission.

Due to the small size of optical waveguide inside OIC or PIC, its operation is difficult to analyze. To resolve this difficulty, numerical methodologies are used to study the wave propagation inside the core of optical waveguides. The Beam Propagation Method (BPM) has been proposed for 3D optical waveguide analysis. The waveguide structure are considered as many frames as along the transverse plane. The analysis are computed and electromagnetic field are updated frame by frame. The Fast Fourier Transform (FFT) technique is

978-1-4673-9749-0/16/\$31.00 ©2016 IEEE

the extension of BPM method, described in [2] for paraxial wave equation (PWE). Nevertheless, the FFT-BPM could not apply to the optical waveguide with large refractive-index variation in longitudinal direction. This limitation has been overwhelmed for the Finite-Difference Method (FDM) [3] and Finite-Element Method (FEM) [4]. Besides, the Pade' approximation operator has been deployed in the numerical technique in order to improve the efficiency of optical waveguide analysis as described in [5]. Moreover, the FE-BPM formulation based on vector wave equation (Vectorial FE-BPM) is also useful for the polarization analysis of the optical wave. FE-BPM with edge-based triangular element are more accurate for modeling arbitrary shape waveguide cross-section [6]. However in PIC or OIC, the optical waveguide always has rectangular shape due to the fabrication process. The formulation of FE-BPM is appropriate for PIC based on node-based rectangular element which can apply to PIC or OIC efficiently. In addition, the application in the wide-angle structure should be formulated by using Padé approximation in order to improve the accuracy of calculation and analysis [7].

In this paper, we demonstrate the simulation results from the numerical method based on FE-VBPM with transverse vectorial H-field formulation and application of node-based rectangular element for the 2D perfectly matched layer (PML) as described in [8]. The objective of this paper is to compare results from the first and second order of Padé approximation. This results will prove the higher order of Padé is more accuracy than lower order for non-uniform structure analysis. The relative power is calculated and analyzed in the tapered waveguide which is the wide-angle structure formulation.

This paper is organized as follows. The FE-VBPM formulation and computation are described in Section II. Then, the simulation settings, results and discussion are presented in Section III. Finally, the Section IV is the conclusion.

## II. FE-BPM FORMUALATION

## A. Wave equations in terms of transverse magnetic field

In this work, a time-harmonic electromagnetic field is considered. The governing equation for electromagnetic wave propagation in lossless dielectric optical waveguide can be derived from the Maxwell's equations for both dielectric medium